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DIGITAL SIGNAL PROCESSING ON BASIS OF GENERALIZED POLYNOMIAL FORMS

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Abstract: The digital signal processing based on representation of discrete function in generalized polynomial form is considered. The synthesis technique of polynomial orthogonal functions in extended base of operations and mixed valued of variables is discussed. The effectiveness of polynomial implementation is obtained and the conditions of repetition-free representation are defined.

1. Introduction

The discrete orthogonal transformation is represented as:

,
$$a(i) = \sum_{i=0}^{K-1} f(t)\,\theta^{-1}(t,\, i), \qquad \sum_{i=0}^{K-1} \theta^{-1}(t,i)\,\theta(i,\tau) = \begin{cases} 1, & t = \tau; \\ 0, & t \neq \tau. \end{cases} \quad (1)$$

where $f(t)$ are digital readouts of a signal in sampling instants $t = \overline{0, K-1}$, $a(i)$ ($i = \overline{0, K-1}$) are a spectrum of the signal, or coefficients of expansion on the base $\left\{\theta(i,t), \theta^{-1}(t,i)\right\}_{i,t=0}^{K-1}$; K is number of readouts in temporal (frequency) area. The expressions (1) also are represented as matrixes equations:

$$F = D \times A, \qquad A = D^{-1} \times F, \qquad D^{-1} \times D = E, \quad (2)$$

where F (A) is a sample (spectrum) of signal, the vector with length K; D (D^{-1}) is matrix of direct (inverse) transformation with dimension $K \times K$; E is unit matrix with the same dimension.

2. Statement of the problem

We shall execute digital signal processing by integer calculations (2). The device of discrete data processing we shall reduce to a implementation of some discrete function f from n variables. The function has k_f of values. The variables also have various values: $x_j \in \left\{0, \ldots, k_j - 1\right\}$ ($j = \overline{0,\ n-1}$), defined by separation of argument t on n parts not equal to each other. In an outcome of synthesis of formal representation of function among all forms we shall search such, which are effective at a implementation.

3. Polynomial forms

Arbitrary discrete function we shall present in the generalized polynomial form [1]:

$$f(X) = \sum_{i=0}^{K-1} a_i \left(c_i \delta_n x_{n-1}^{i_{n-1}} \delta_{n-1} \ldots \delta_0 x_0^{i_0}\right) = \sum_{i=0}^{K-1} a_i \theta(i, X), \quad (3)$$

where $K = k_{n-1} \ldots k_1 k_0$; a_i are coefficients of form; c_i are arbitrary constants; δ_j are logic operations included in a set $\Delta = \left\{\delta_n, \delta_{n-1}, \ldots, \delta_1, \delta_0\right\}$; $x_j^{i_j}$ is variable x_j in a logic degree i_j, $x_j^{i_j} = x_j \gamma_j i_j$, all degree operations belong to a set $\Gamma = \left\{\gamma_{n-1}, \ldots, \gamma_1, \gamma_0\right\}$; i_j are digits of i in a positional representation with a mixed bases: $i = \left(i_{n-1}, \ldots i_0\right)_K$, $i_j \in \left\{0, \ldots, k_j - 1\right\}$; $\theta_i(X)$ are polynomial orthogonal functions. The operations of addition and multiplication belong to algebra $B = \langle P, +, \cdot \rangle$, given on a set $P = \left\{0, 1, \ldots, k_f - 1\right\}$.

4. Synthesis of polynomial forms

For synthesis of the form (3) is used discrete orthogonal transformation (2). The square matrixes D and D^{-1} may be obtained as follows [2]. We set a kernel of transformation, which defines matrixes of degree operations W_j with elements $w_j(\alpha, \beta) = \alpha \gamma_j \beta$ ($\alpha, \beta = \overline{0, k_j - 1}$), where α (β) is a row (column) number.

The rows (columns) of the matrixes W_j should be linearly independent in B. We create the matrix D by use a recurrent rule: $G_0 = W_0$, $G_{j+1} = W_j \otimes_j G_j$ $\left(j = \overline{0, n-2}\right)$, $D = C\delta_n G_{n-1}$, where $\otimes_j$ is generalized Kronecker product of matrixes; C is a matrix consisting of K identical rows of K arbitrary constants. A choice of operations and their sequences should not conduct to a linear dependence of rows (columns), and constants c_i we set so, that the determinant of D should be different from zero. The matrix D^{-1} is calculated from a orthogonal condition of (2). A inversion of matrixes as well as the check of linear independence of rows (columns) we process in B. The generalized Kronecker product of matrixes is defined as: $W_j \otimes_j G_j = \left[w_j(\alpha,\beta)\delta_j G_j\right]_{\alpha,\beta=0}^{k_j-1}$, the result of which is a matrix consisting of $k_j \times k_j$ submatrixes G_j transformed by operation δ_j with the first operand $w_j(\alpha,\beta)$. At the synthesis of the form (3) a check of a functional completeness of base $\Omega = \{+, \cdot\} \cup \Delta \cup \Gamma$ is reduced to the check of linear independence of rows (columns) of the matrix G_{n-1} in B.

5. Criterions of effectiveness

An information capacity of the polynomial names value

$$I = \sum_{i=0}^{K-1} log_2\left(a_i + 1\right), \text{ [bit]}.$$

An information entropy names value $H = I/K$, [bit per coef.]. Effectiveness of representation (implementation) of function we shall define relatively a tabulared mode of computing, which is invariant to a choice of a function. Effectiveness on time ξ we shall define as a ratio of time τ_t of the tabulared computing to time of computing by polynomial forms τ: $\xi = \tau_t/\tau$. Space effectiveness η we shall define as a ratio of information capacity of a truth table I_t to information capacity of a polynomial I: $\eta = I_t/I = K\,log_2 k_t/I = log_2 k_f/H$. Complex effectiveness ϑ we shall understand as a product of effectiveness on time ξ and space effectiveness η: $\vartheta = \xi\eta = \tau_t\, log_2 k_f/\tau H$. Form is effective when $\vartheta > 1$.

6. Information estimate

The information capacity of a truth table I_t sets an amount of degree of freedoms, which are used at the representation of arbitrary function. The form should save an information capacity of a truth table: $I_p \geq I_t$, where I_p is information capacity of form (3) which have M of nonzero coefficients. Let k_j and k_f is equal k, then

$$I_p = \left(M + (n-1)k^2 + (n+1)k\right) log_2 k \geq k^n\, log_2 k,$$

whence we obtain an estimate for effectiveness of the form:

$$\vartheta \leq \frac{\tau_t k^n}{2(n+1)\left(k^n - (n-1)k^2 - (n+1)k\right)^2}, \qquad k^n > (n-1)k^2 + (n+1)k \tag{4}$$

When $k^n \leq (n-1)k^2 + (n+1)k$ the repetition-free expression is synthesized on base $\Delta \cup \Gamma$:

$$f(X) = c\delta_n x_{n-1}^{i_{n-1}} \delta_{n-1} \ldots \delta_1 x_1^{i_1} \delta_0 x_0^{i_0}, \tag{5}$$

The effectiveness of the repetition-free form (5) is $\tau_t k^n/2n$. From (4) issue, when n and k are large almost all coefficients are different from zero and almost all functions are realized with complexity close to maximum. It does not hinder to receive the effective forms, but only for a limited class of functions.

7. Conclusion

The described technique of synthesis of polynomial forms allows to receive untraditional methods of digital signal processing when the use of microprocessors may be intensified.

References

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