

Spectral Methods in Logical Data Analysis

V. S. Vykhoanets

Pridnestrovsk State University, Tiraspol, Moldova

Received November 28, 2000

Abstract—A discrete function of arguments of different digits is represented as a formula in a functionally complete base of operations. Bases consisting of two algebraic operations, a fixed set of functions, and functions from a given class are studied. Such a representation is related to spectral expansions. A synthesis method for polynomial and nonpolynomial forms is designed. Information and asymptotic estimates of the complexities of formulas are determined.

1. INTRODUCTION

Logical data processing methods are widely employed in modern computing devices. They also find extensive application in logical control, digital signal processing, pattern recognition, and in designing discrete devices, etc.

There are two extreme approaches to logical computation. The first is used if input data are transformed into output through nonregular representations, whereas the second is used if computations are carried out with regular (spectral) forms.

Nonregular forms are obtained through decomposition of a general-type discrete function. For example, a function f of n variables $X = \{x_0, x_1, \dots, x_{n-1}\}$ at every decomposition step is expressed as a function $f(X) = g(h(Y), Z)$ of other functions g and h , where Y and Z are subsets of X [1]. There are single, multiple, iterative, and recursive decompositions. Decomposition terminates upon generation of a representation that is realizable with the capabilities of the computing system.

General-type decomposition is regarded as a combinatorial problem involving the search for a large number of solutions and this is virtually nonrealizable if the dimension of the problem is large [2]. As a rule, nonregular processing is represented in an algorithmic form. The theoretical base underlying this representation are well-known paradigms of analysis of subject domain and design of program tools, namely, logical, functional, structural, and object-oriented methods.

Decision diagrams [3], which are essentially a graphic form of representation of logical programs [4], occupy an intermediate position between nonregular and regular forms. This type of representation is based on the decomposition of a discrete function into an algebraic system formed by two operations, referred to as addition and multiplication. Decomposition is implemented by functions θ_i of one or several variables Y : $f(X) = \sum \theta_i(Y) \times a_i(Z)$, where a_i are functions (expansion coefficients) dependent on the remaining variables Z . In the next step, the functions a_i (θ_i) are decomposed, possibly, in a different algebraic system and by other functions. This process is repeated until expansion coefficients become constants.

In the extreme case, regular forms are based on expansion—spectral representation of a discrete function for $Y = X$ and $Z = \emptyset$:

$$f(X) = \sum_{i=0}^{m-1} \theta_i(X) \times a_i, \quad (1)$$

where a_i are constants or coefficients of expansion in the base $\Omega = \{+, \cdot, \theta_i (i = \overline{0, m-1})\}$ and θ_i are spectral functions.