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ADDITEVE ALGEBRA FOR DIGITAL SIGNAL PROCESSING

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Abstract. A digital signal processing based on a representation over additive algebra is discussed. This system can be used for syntheses various spectral representations of digital readouts of a signal. Theorem of spectral decomposition is formulated. The given theorem allows constructing spectral functions for various implementations. Example of the digital signal processing is given.

1. INTRODUCTION

Image analysis, signal processing, logic design are normally thought of in terms of multiple-valued signals; however it is natural to think of variables with symbolic or integer values. For that a multiple-valued signal f is transformed into the spectral representation by the discrete transformation

$$f(t)=\sum_{i=0}^{m-1}\theta(t,i)\cdot a(i),\qquad a(i)=\sum_{t=0}^{m-1}\vartheta(i,t)\cdot f(t),\qquad \sum_{i=0}^{m-1}\theta(t,i)\cdot\vartheta(i,\tau)=\begin{cases}1, t=\tau\\ 0, t\neq\tau\end{cases}, \tag{1}$$

where $f(t)$ are digital readouts of a signal in sampling instants $t=\overline{0,m-1}$; $a(i)$ ($i=\overline{0,m-1}$) is a spectrum of the signal; $\theta(t,i)$ and $\vartheta(i,t)$ are a system of orthogonal signals (functions); m is a number of readouts in the temporal and the spectral (frequency) area.

The expressions (1) also can be written as a matrix equation $F=D\times A$ (if there exist Q such that $A=Q\times F$, $Q\times D=E$ we have orthogonal transformation), where F (A) is a m-vector, D (Q, E) is a direct (inverse, unit) $m\times m$-matrix.

There are a few algebraic systems [1], logic algebra, additive and multiplicative algebra, Galois field and ring of integers, which allow finding a_i from (1). Galois field, ring of integers and field of real numbers are well known [2-4], but additive algebra needs to research. In this paper additive algebraic system for digital signal processing is considered.

2. ADDITIVE ALGEBRA

Definition 1. Let a domain N_k is a finite set of integers $\{0,1,\dots,k-1\}$.

Definition 2. Let $R_L=\langle N_k,+,\cdot\rangle$ be an additive algebra and there exists $\sigma\in N_k$ and $\iota\in N_k$ ($\iota\neq\sigma$) such that $a+\sigma=a$, $\sigma+a=a$ and $\sigma\cdot a=\sigma$, $\iota\cdot a=\iota$ for all $a\in N_k$. Element σ is called zero and element ι is called unit. In addition let $G_A=\langle N_k,+\rangle$ is a commutative group.

Definition 3. The cyclic order of element $a\in G_A$ is a minimal whole number $c_a>0$, such that cyclic sum $c_a\circ a=\underbrace{a+a+\dots+a}_{c_a}=\sigma$, where σ is an identity element of G_A. Let $0\circ a=\sigma$ and let $(-\lambda)\circ a=\lambda\circ(-a)$ where λ is an integer.

Definition 4. The cyclic order of group G_A is a minimal order of its elements except σ.

Lemma. Equation $\lambda\circ a=b$ has unique solution for all $a,b\in G_A$ if and only if $|\lambda|<c$, where c is a cyclic order of commutative group G_A.

Definition 5. A logical matrix L_m is a $m\times m$-matrix; each of whose elements is zero or unit. If we replace the elements of L_m with 0 and 1 respectively, we find matrix $\widehat{L}_m$. The matrix $\widehat{L}_m$ is called a conjugate matrix of L_m.

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Theorem. Any function f can be represented in the form (1) over $\boldsymbol{R_A}$ if $\boldsymbol{D}$ is a logical matrix $\boldsymbol{L_m}$ and if the modulo of determinant of conjugate matrix $\widehat{\boldsymbol{L}}_{\boldsymbol{m}}$ less then a cyclic order of group $\boldsymbol{G_A}$. Then $\Delta \circ \boldsymbol{A} = \overline{\boldsymbol{D}}^{\boldsymbol{T}} \circ \boldsymbol{F}$ where $\overline{\boldsymbol{D}}$ is an algebraic complement $\widehat{\boldsymbol{L}}_{\boldsymbol{m}}$, Δ is a determinant of $\widehat{\boldsymbol{L}}_{\boldsymbol{m}}$.

3. DEMONSTRATION EXAMPLE

Let a addition and a multiplication are operations defined by matrix $\boldsymbol{S}$ and $\boldsymbol{P}$ such that $s_{ij} = i + j$ and $p_{ij} = i \cdot j$ ($i, j = \overline{0, m-1}$),

$$\boldsymbol{S} = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 3 & 2 \\ 2 & 3 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{bmatrix}; \ \boldsymbol{P} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & * & * & * \\ * & * & * & * \\ 0 & 1 & 2 & 3 \end{bmatrix},$$

where * is an indifference number. A cyclic order of G_A is equal to 2. Obviously $\sigma = 0$ and $\iota = 3$. Note the addition is the digit-to-digit nonequivalence and the multiplication can be realized as digit-to-digit conjunction, for example. Let a spectral system is defined as the following matrixes:

$$\boldsymbol{D} = \begin{bmatrix} 3 & 0 & 0 & 0 & 0 & 0 \\ 3 & 3 & 0 & 0 & 0 & 0 \\ 3 & 3 & 3 & 0 & 0 & 0 \\ 0 & 3 & 3 & 3 & 0 & 0 \\ 0 & 0 & 3 & 3 & 3 & 0 \\ 0 & 0 & 0 & 3 & 3 & 3 \end{bmatrix}, \ \overline{\boldsymbol{D}}^{\boldsymbol{T}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 1 & 0 & -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & -1 & 1 & 0 \\ 0 & -1 & 1 & 0 & -1 & 1 \end{bmatrix}, \ \Delta = 1.$$

A direct (inverse) spectral function is a column of matrix $\boldsymbol{D}$ ($\overline{\boldsymbol{D}}^{\boldsymbol{T}}$). These spectral functions can be got with a logical shifting of bit strings. Note the first two thirds of they are phase-sensitive functions, the others are phase-insensitive functions.

Then for a signal $\boldsymbol{F} = [1, 3, 0, 2, 1, 0]^T$ we have $\boldsymbol{A} = \overline{\boldsymbol{D}}^{\boldsymbol{T}} \circ \boldsymbol{F} = [1, 2, 3, 3, 1, 2]^T$. After low-pass filtering we have $\tilde{\boldsymbol{A}} = [1, 2, 3, 3, 0, 0]^T$ and $\tilde{\boldsymbol{F}} = [1, 3, 0, 2, 0, 3]^T$, where $\tilde{\boldsymbol{F}}$ is a phase-sensitive constituent of the current signal $\boldsymbol{F}$.

CONCLUSION

The additive algebra as a alternative algebraic system for digital signal processing provides some advantages in comparison with such the algebraic systems as Galois field, ring of integers and field of real numbers.

In particular if we use spectral bases over the additive algebra then spectral functions are easy to realize, as they are bimodal. For execution of discrete transformations we need simple computing facilities with low operation speed and small digit capacity as it is not required to use a multiplication for integers or real numbers.

At last the digital signal processing over additive algebra preserves all helpful properties of the discrete orthogonal transformation.

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