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HAAR-LIKE SYSTEMS OF SIGNALS

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Abstract. A problem of generation of bases having regular and homogeneous analytical construction of spectral functions is considered. Synthesis of analytical construction, where Haar-like and truncated systems of signals is a particular case, is discussed.

Let us expand the function f depended on variables $x_0^{[n]} = (x_0, x_1, \ldots, x_{n-1})$ with k_0, k_1, …, k_{n-1} digits correspondingly in a system of spectral functions $\phi_i^{(t)}$ by the last $s+1$ variables $x_t^{[s+1]} = (x_t, x_{t+1}, \ldots, x_{t+s})$,

$$f(x_0^{[n]}) = \sum_{i=0}^{k_t^{[s+1]}-1} \phi_i^{(t)}(x_t^{[s+1]}) \times a_i^{(t)}(x_0^{[t]}),$$

where t is an index of first variable, here $t = n - s - 1$. For all $y, z \in N_{k_{t+s}}$ let $\phi_i^{(t)}(x_t^{[s]}, y) = \phi_i^{(t)}(x_t^{[s]}, z)$ $(i = \overline{0, k_t^{[s]} - 1})$, where $k_t^{[s]} = k_t k_{t+1} \ldots k_{t+s-1}$, i.e., the first $k_t^{[s]}$ out of $k_t^{[s+1]}$ spectral functions $\phi_i^{(t)}$ do not depend on the variable x_{t+s}. This is possible in all algebras, except in the multiplicative algebra and the algebra of logic [1, 2]. Then we can write

$$f(x_0^{[n]}) = f^{(t)}(x_0^{[t+s]}) + \sum_{i=k_t^{[s]}}^{k_t^{[s+1]}-1} \phi_i^{(t)}(x_t^{[s+1]}) \times a_i^{(t)}(x_0^{[t]}),$$

where $f^{(t)}$ is a function of $t+s$ variables. In turn, expanding $f^{(t)}$ by the last $s+1$ variables, we obtain

$$f(x_0^{[n]}) = \sum_{i=0}^{k_0^{[s]}-1} \phi_i(x_0^{[s-1]}) \times a_i + \sum_{t=0}^{n-s-1} \sum_{i=k_t^{[s]}}^{k_t^{[s+1]}-1} \phi_i^{(t)}(x_t^{[s+1]}) \times a_i^{(t)}(x_0^{[t]}),$$

Since the coefficients $a_i^{(t)}$ are functions, we represent them as expansions in systems of orthogonal functions $\psi_{ij}^{(t)}$

$$f(x_0^{[n]}) = \sum_{i=0}^{k_0^{[s]}-1} \phi_i(x_0^{[s-1]}) \times a_i + \sum_{t=0}^{n-s-1} \sum_{i=k_t^{[s]}}^{k_t^{[s+1]}-1} \phi_i^{(t)}(x_t^{[s+1]}) \times \left\{ \sum_{j=0}^{k_0^{[t]}-1} \psi_{ij}^{(t)}(x_0^{[t]}) \times a_{ij}^{(t)} \right\},$$

where for the sake of definiteness we take $k_t^{[0]} = 1$ and $\psi_{ij}^{(0)} = 1$. We determine the transformation matrices $D = D_{n-s}$ and $Q = Q_{n-s}$ from the recurrent equations ($t = \overline{0, n-s-1}$),

$$D_{t+1} = \left(E_{k_0^{[t]}} \otimes \Phi^{(t)} \right) \times \begin{bmatrix} D_t & \varnothing & \cdots & \varnothing \\ \varnothing & \Psi_1^{(t)} & \cdots & \varnothing \\ \vdots & \vdots & \ddots & \vdots \\ \varnothing & \varnothing & \cdots & \Psi_{k_t-1}^{(t)} \end{bmatrix}, \quad Q_{t+1} = \begin{bmatrix} Q_t & \varnothing & \cdots & \varnothing \\ \varnothing & \mu\Psi_1^{(t)} & \cdots & \varnothing \\ \vdots & \vdots & \ddots & \vdots \\ \varnothing & \varnothing & \cdots & \mu\Psi_{k_t-1}^{(t)} \end{bmatrix} \times \left(E_{k_0^{[t]}} \otimes \mu\Phi^{(t)} \right),$$

under the initial conditions $D_0 = \Phi$, $Q_0 = \mu\Phi$ and $\Psi_i^{(0)} = E_{k_0^{[s]}}$, where $\varnothing$ is a square matrix of size $k_0^{[t]}$, E_k is unity matrix of size k, μ denotes an inversion of matrix, $\otimes$ denotes Kronecker product of matrices, $\Phi = [\phi_i(j)]$ ($i, j = \overline{0, k_0^{[s]} - 1}$), $\Phi^{(t)} = \left[\phi_i^{(t)}(j)\right]$ ($i, j = \overline{0, k_t^{[s+1]} - 1}$), $\Psi_i^{(t)} = \left[\psi_{ij}^{(t)}(l)\right]$ ($i = \overline{0, k_t - 1}$, $j, l = \overline{0, k_0^{[t]} - 1}$).

So, we obtain the analytical constriction that describes many spectral bases. The construction depends on scale or depth of transformation s. When $s = 0$ we have Haar-like or truncated spectral functions [3]. Arbitrary choice of kernel of transformation Φ, systems of function $\Phi^{(t)}$ and $\Psi_i^{(t)}$ allow us to get many systems of signals, which have regular and homogeneous analytical construction.

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