Automation and Remote Control, Vol. 65, No. 12, 2004, pp. 1861–1871. Translated from Avtomatika i Telemekhanika, No. 12, 2004, pp. 3–14.

Polynomial Factorization of Spectral Bases

V. S. Vykhovanets

Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia
Received May 25, 2004

Abstract—Polynomial factorization of spectral bases is studied, expressing polynomial factorization as the representation of a system of spectral functions defined by an integral discrete transformation matrix in the form of the Kronecker product of matrices of reduced dimension. Such a representation is helpful in expressing an ordered system of functions by a unified formula in a base of binary operations. An algorithm for polynomial factorization of matrices, its theoretical principles, and results of an experiment are presented.

1. INTRODUCTION

Factorization of the spectral base is traditionally interpreted as the representation of a system of spectral functions defined by a direct (inverse) transformation matrix as a product of weakly filled matrices of reduced dimension [1]. Factorization is essential for implementing fast spectral transformations containing multiplication of a matrix by a vector.

Factorization results in a fast transformation procedure, in which like terms are reduced due to the distributivity of multiplication relative to addition and elimination of operations in trivial cases when multiplication by zero and 1, and addition with zero are not implemented.

Reduced number of operations is the first necessary condition for factorization. The second condition is the representation of the transformation algorithm when there is a need for such a representation in which operations occur in a regular order in the algorithm. Regular order of occurrence not only reduces computation time, but also simplifies the algorithm and enhances its effectiveness.

Thus, the aim of the traditional approach to factorization is to design effective spectral transformation procedures. Here no attention is paid to the effectiveness of computation of functions (matrices), which are assumed to be given.

In this paper, we study polynomial factorization, i.e., representation of a discrete transformation matrix as a Kronecker product of matrices of reduced dimension. In our approach, we determine matrices and operations involved in the Kronecker product for which a formula for a system of spectral functions can be found as a result of factorization and the initial matrix of a discrete transformation is computed.

2. FORMULATION OF THE PROBLEM

To factorize a given $m \times m$ integer matrix $\mathbf{D}$, let us find matrices $\mathbf{P}_j$ and $\mathbf{O}_t$ ($j = \overline{0, n-1}$, $t = \overline{0, n-2}$) such that the equality

$$\mathbf{D} = \mathbf{P}_0 \otimes_0 \mathbf{P}_1 \otimes_1 \ldots \otimes_{n-2} \mathbf{P}_{n-1} = \mathop{\otimes}_{i=0}^{n-1} \mathbf{P}_i \tag{1}$$

holds within to the placement of braces, where the elements and size of $\mathbf{P}_j$ are not greater than some number k_p and the Kronecker product $\otimes_t$ (see Definition 2 below) is implemented for binary operations defined by matrices $\mathbf{O}_t$, whose elements and size are also not greater than k_p.