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DETERMINATE SYSTEMS

Algebraic Decomposition of Discrete Functions

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Abstract—Consideration was given to the functional decomposition of the discrete systems which is reducible to the functional decomposition of the discrete functions, where by the decomposition is meant the representation of a function by a formula in the basis of unary and binary operations. The algebraic decomposition in an algebra consisting of two binary operations and functions of two variables was studied. A procedure of formula design on the basis of composition of repetition-free subformulas was substantiated. Both exact and asymptotic complexity estimates of the designed formulas were given.

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1. INTRODUCTION

Decomposition of functions is one of the universal approaches to various scientific and engineering problems. The function is represented as a composition of smaller-dimensionality functions. In the limit required for practical purposes, the composition is constructed of the variables and operations which are immediately realized by the computing facility.

Decomposition of discrete functions is concerned mostly with the decomposition of Boolean functions defined over two-element sets. The fundamental book of G. Boole [1] (1854) ushered in the theory of decomposition. After a long period, I.I. Zhegalkin published in 1927 a paper [2] on transformations of the Boolean functions which laid foundation to the modern methods of orthogonal transformations. The dawn of the era of applied logic design is related with the names of C.E. Shannon [3, 4] (1938, 1949) and M.A. Gavrilov [5, 6] (1945, 1946).

The first results on tabular decomposition of functions by disjunction of variables were obtained by R.L. Ashenhurst [7] (1952) and W.L. Semon [8] (1952). G.N. Povarov [9] (1954) described the disjunctive decomposition in analytical terms. In extension of the Zhegalkin's results, L.S. Reed [10] (1954) and D.E. Muller [11] (1954) introduced the notion of analytical construction and employed decomposition of functions in a form known today as the canonical Reed–Muller form. Multiple decomposition was studied by H.A. Curtis [12] (1958), a concept of differentiation of the Boolean functions was suggested by S.B. Akers [13] (1959), and joint minimization of the systems of Boolean functions under disjunctive decomposition was put into practice by J.P. Roth and R.M. Karp [14, 15] (1960, 1962).

The works of A.V. Kuznetsov on repetition-free decomposition [16] (1958), E.I. Nechiporuk on linear transformations of variables at minimization [17] (1958), and K.K. Maitra on cascaded decomposition [18] (1962) were devoted the limited classes of discrete functions and special cases of decomposition. Representation of functions by the decision diagrams was considered by C.Y. Lee [19] (1959) and S.B. Akers [20] (1978). The spectral forms were used by M.G. Karpovskii and E.S. Moskalev [21] (1970), and the method of spectral expansion-based decomposition was employed by R.J. Lechner [22] (1971).