Optimal Spectral Expansion for Discrete Control

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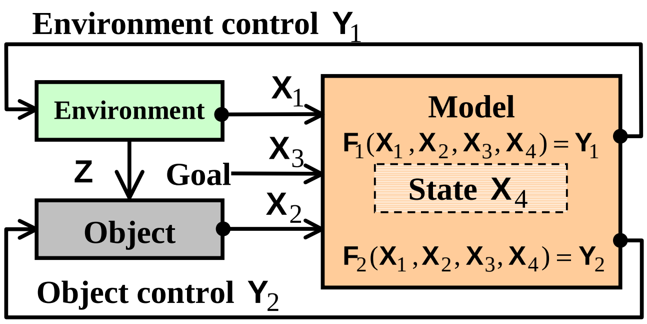
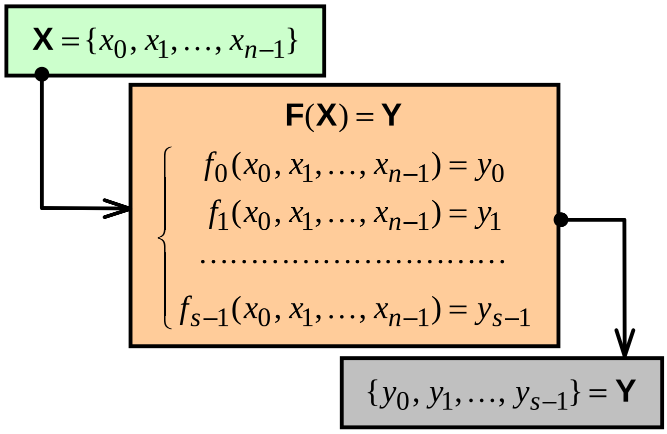
1. Preliminary – Discrete Control



2. Problem – Discrete Decomposition

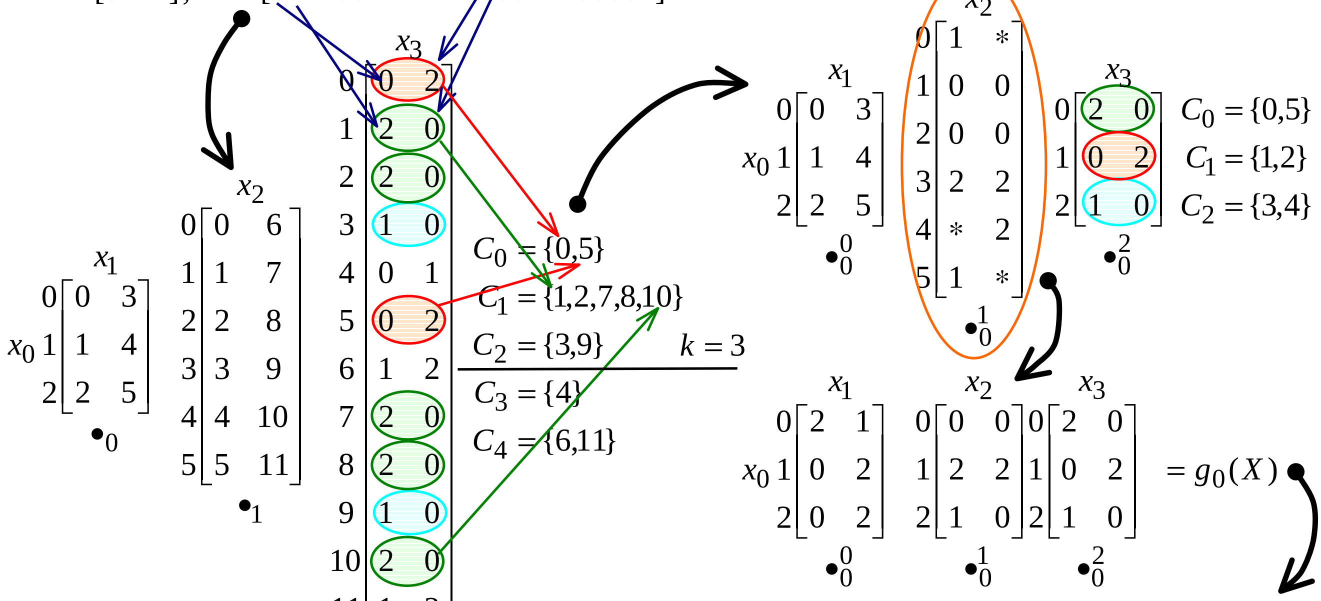


3. Approach – Spectral expansion

$$\left.\begin{aligned} f(X) &= \sum_{i=0}^{m-1} g_i(X) \times h_i \\ g_i(X) &= x_0 \bullet_i^0 x_1 \bullet_i^0 \dots \bullet_i^{n-2} x_{n-1} \end{aligned}\right\} \to \left\{\begin{aligned} & f(X) = \sum_{i=0}^{M-1} g_i(X), \\ & \text{var } \bullet_i^j \to \min M \end{aligned}\right.$$

4. Solution – Analytical synthesis

$\mathbf{K} = [3222]$, $\mathbf{F} = [022100122121200012200002]$



$\mathbf{G}_0 = [022100122121200012200002]$

$\mathbf{F}_1 = \mathbf{F} - \mathbf{G}_0$, and so on…

5. Result – Formula over binary operations

$\mathbf{F} = \mathbf{G}_0 + \mathbf{G}_1$ or $f(X) = g_0(X) + g_1(X)$

$$f(X) = x_0 \bullet_0^0 x_1 \bullet_0^1 x_2 \bullet_0^2 x_3 + x_0 \bullet_1^0 x_1 \bullet_1^1 x_2 \bullet_1^2 x_3$$

$$\bullet_0^0 = \begin{bmatrix} 2 & 1 \\ 0 & 2 \\ 0 & 2 \end{bmatrix}, \quad \bullet_0^1 = \begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 1 & 0 \end{bmatrix}, \quad \bullet_0^2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 1 & 0 \end{bmatrix}, \qquad \bullet_1^0 = \begin{bmatrix} 1 & 0 \\ 0 & 2 \\ 0 & 1 \end{bmatrix}, \quad \bullet_1^1 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 2 & 0 \end{bmatrix}, \quad \bullet_1^2 = \begin{bmatrix} 0 & 0 \\ 2 & 2 \\ 0 & 2 \end{bmatrix}.$$

6. Efficiency – Better than tabular realization

$$E \sim (k - \alpha_k)\bar{k} > 1, \quad 0 < \alpha_k < 1$$